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THE MATHEMATICS TEACHER

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ELECTIVE COURSES IN MATHEMATICS FOR SECONDARY SCHOOLS.

A PRELIMINARY REPORT BY
THE NATIONAL COMMITTEE ON MATHEMATICAL REQUIREMENTS.

INTRODUCTION.

The Committee has elsewhere expressed its judgment that in the seventh, eighth and ninth grades mathematics should be a required subject. In the tenth, eleventh and twelfth grades, however, the extent to which election of subjects is permitted will depend on so many factors of a general character that it seems unnecessary and inexpedient for the present Committee to urge a positive requirement beyond the minimum one for the seventh, eighth and ninth grades. The subject must, like others, stand or fall on its intrinsic merit or on the estimate of such merit by the authorities responsible at a given time and place. The Committee believes, nevertheless, that every standard high school should not merely offer courses in mathematics for the tenth, eleventh and twelfth grades, but should encourage a large proportion of the pupils in its general courses to take some or all of these courses. Apart from the intrinsic interest and great educational value of the study of mathematics, it will in general be necessary for those preparing to enter college or to engage in the numerous occupations involving the use of mathematics to do work beyond the minimum requirement.

The present report is intended to suggest the most valuable mathematical training for students in general courses in secondary schools that will be supplementary to the first courses in mathematics as outlined in a previous report of the National

Committee.* Under present conditions most of this work will normally fall in the last two years of the high school, *i.e.*, in general, in the eleventh and twelfth school years.

The selection of material is based on the following general principles:†

1. The primary purpose of the teaching of mathematics should be to develop those powers of understanding and analyzing relations of quantity and of space which are necessary to a better appreciation of the progress of civilization and a better understanding of life and the universe about us and to develop those habits of thinking which will make these powers effective in the life of the individual.

2. The courses in each year should be so planned as to give the pupil the most valuable mathematical information and training which he is capable of receiving in that year with provision for his vocational and later educational needs.

The second principle leads us to the conclusion that the material for the elective courses offered should include, as far as possible, those mathematical ideas and processes that have the most important applications in the modern world. As a result we will naturally include certain material that at present is not ordinarily given in secondary courses; as, for instance, the material concerning the calculus. On the other hand we shall exclude certain other material that is now included in college entrance requirements. The results of an investigation made by the National Committee in connection with a study of these requirements indicates that modifications to meet these changes will be desirable from the standpoint of both college and secondary school.*

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* Cf. Secondary School Circular No. 5 (February, 1920), U. S. Bureau of Education.

† The first principle is adopted unchanged from the report just referred to; the second principle is a modified form of the second principle of the same report. This modification is a logical consequence of the somewhat different aim of elective courses in mathematics as compared with courses for all students; for after students have completed the minimum essentials of mathematics it will be found desirable to differentiate their training according to special life aims, interests or aptitudes in so far as these are discoverable.

* Cf. "College Entrance Requirements in Mathematics," a preliminary report, to appear in the May, 1921, number of the MATHEMATICS TEACHER.

One cannot too strongly emphasize the fact that the broadening of content of high school courses in mathematics suggested in the report on the first courses and in the present report will very materially increase the usefulness of these courses to those who pursue them. It is of prime importance that educational administrators and others charged with the advising of students should take careful account of this fact in estimating the relative importance of mathematical courses and alternative elections. The number of important applications of mathematics in the activities of the world is today very large and is increasing at a rapid rate. This aspect of the progress of civilization has been noted by all observers who have combined a considerable knowledge of mathematics with an alert interest in the newer developments in other fields. It was revealed in very illuminating fashion during the recent war by the insistent demand for persons with varying degrees of mathematical training for many war activities of the first moment. Other forms of special training were also in demand, but in no single instance was the demand so widespread. If the same effort were made in time of peace to secure the highest level of efficiency available for the specific tasks of modern life, the demand for those trained in mathematics would be no less insistent. For it is in no wise true that the applications of mathematics in modern warfare are relatively more important or more numerous than its applications in those fields of human endeavor which are of a constructive nature.

There is another important point to be kept in mind in considering the relative value to the average student of mathematical courses and various alternative courses. If the student who omits the mathematical courses has need of them later, it is almost invariably more difficult and it is frequently impossible for him to obtain the training in which he is deficient. In the case of a considerable number of alternative subjects a proper amount of reading in spare hours at a more mature age will ordinarily furnish him the approximate equivalent to what he would have obtained in the way of information in a high school course in the same subject. It is not, however, possible to make up deficiencies in mathematical training in so simple a fashion. It requires systematic work under a competent teacher to master properly the technique of the subject and any break in the continuity of the work is a handicap for which increased maturity rarely compensates. Moreover, when the individual discovers his need for further mathematical training, it is usually difficult for him to find the time from his other activities for systematic work in elementary mathematics.

RECOMMENDATIONS FOR ELECTIVE COURSES.

The following topics are recommended for inclusion in the mathematical offerings to pupils who have satisfactorily completed the work outlined in the National Committee's report on The Reorganization of the First Courses in Secondary School Mathematics, comprising the elementary notions of Algebra, Intuitive Geometry, Numerical Trigonometry and Demonstrative Geometry.

1. **Algebra.** (*a*) *Simple functions of one variable.* Numerous illustrations and problems involving linear, quadratic and other simple functions including formulas from science and common life. More difficult problems in variation than those included in the earlier course.

(*b*) *Equations in one unknown.* Various methods for solving a quadratic equation (such as factoring, completing the square, use of formula) should be given. In connection with the treatment of the quadratic a very brief discussion of complex numbers should be included. Simple cases of the graphic solution of equations of degree higher than the second should be discussed and applied.

(*c*) *Equations in two or three unknowns.* The algebraic solution of linear equation in two or three unknowns and the graphic solution of linear equations in two unknowns should be given. The graphic and algebraic solution of a linear and a quadratic equation and of two quadratics that contain no first degree term and no xy term should be included.

(*d*) *Exponents, radicals and logarithms.* The definitions of negative, zero and fractional exponents should be given and it should be made clear that these definitions must be adopted if we wish such exponents to conform to the laws for positive integral exponents. Reduction of radical expressions to those involving fractional exponents should be given as well as the inverse transformation. The rules for performing the fundamental operations on expressions involving radicals, and such transformations as

$$\sqrt[n]{a/b} = \frac{1}{b}\sqrt[n]{ab^{n-1}}, \quad \sqrt[n]{a^n b} = a\sqrt[n]{b}, \quad \frac{a}{\sqrt{b} + \sqrt{c}} = \frac{a(\sqrt{b} - \sqrt{c})}{b - c}$$

should be included. In close connection with the work on exponents and radicals there should be given as much of the theory of logarithms as is involved in their application to computation and sufficient practice in their use in computation to impart a fair degree of facility.

(*e*) *Arithmetic and Geometric Progressions.* The formulas for the *n*th term and the sum of *n* terms should be derived and applied to significant problems.

(*f*) *Binominal Theorem.* A proof for positive integral exponents should be given; it may also be stated that the formula applies to the case of negative and fractional exponents under suitable restrictions, and the problems may include the use of the formula in these cases as well as in the case of positive integral exponents.

2. **Solid Geometry.** The aim of the work in solid geometry should be to exercise further the spatial imagination of the student and to give him a knowledge of the fundamental spatial relationships and power to work with them. For many of the practical applications of mathematics it is of fundamental importance to have accurate space perceptions. Hence it would seem wise to have at least some of the work in solid geometry come as early as possible in the mathematical courses, preferably not later than the beginning of the third year of high school. For schools that can complete more than the preliminary courses outlined in the previous report during the first two years it would seem that the more elementary notions of solid geometry might well be listed among the optional topics of the first two years to be studied in connection with related ideas of plane geometry.

The work in solid geometry should include numerous exercises in computation based on the formulas established. This will serve to correlate the work with arithmetic and algebra and to furnish practice in computation. It is of the first importance that such practice be continued throughout the entire mathematical course. For a minimum course it will be possible to omit a considerable number of propositions ordinarily given and many of the exercises of a more theoretical nature. For example, proofs of the more difficult propositions dealing with volumes and areas, that can be established more readily by the methods of the calculus, may well be postponed until they can be discussed as applications of the latter subject. The minimum course should certainly include propositions dealing with perpendiculars to planes, dihedral angles, and the simpler theorems on areas and volumes. It should be possible to complete such a minimum course in a third of a year.*

3. **Trigonometry.** The work in elementary trigonometry begun in the earlier years should be completed by including the logarithmic solution of right and oblique triangles, radian measure, graphs of trigonometric functions, the derivation of the fundamental relations between the functions and their use in providing identities and in solving easy trigonometric equations. There should be, wherever practicable, some use of the transit in connection with the simpler operations of surveying and of the sextant for some of the simpler astronomical observations, such as those involved in finding local time. When no transit or sextant is available, simple apparatus for measuring angles roughly may and should be improvised. Drawings to scale should form an essential part of the numerical work in trigonometry. The use of the slide rule in computations requiring only three place accuracy and in checking other computations is also recommended.

4. **Elementary statistics.** Continuation of the earlier work to include the meaning and use of fundamental concepts and simple frequency distributions with graphic representations of various kinds and measures of central tendency.

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* Some European schools have found it desirable to replace some of the work now usually given in solid geometry by certain important topics of descriptive geometry. Since no textbook is at present available for this purpose the Committee refrains from any recommendation in this direction. The possibility of a scientific and logical treatment of descriptive geometry would seem to be worthy of the attention of teachers, however.

5. **Elementary calculus.** The work should include:

(*a*) The general notion of a derivative as a limit indispensable for the accurate expression of such fundamental quantities as velocity of a moving body or slope of a curve.

(*b*) Applications of derivatives to easy problems in rates and in maxima and minima.

(*c*) Simple cases of inverse problems; e.g., finding distance from velocity, etc.

(*d*) Approximate methods of summation leading up to integration as a powerful method of summation.

(*e*) Applications to simple cases of motion, area, volume and pressure.

The work in calculus should be largely graphic and closely related to that in physics; the necessary technique should be reduced to a minimum by basing it wholly or mainly on algebraic polynomials. No formal study of analytic geometry need be presupposed beyond the plotting of simple graphs.

It is important to bear in mind that while the elementary calculus is sufficiently easy and interesting to justify its introduction special pains should be taken to guard against any lack of thoroughness in the fundamentals of algebra and geometry. No possible gain could compensate for a real sacrifice of such thoroughness.

It should also be borne in mind that the suggestion of including elementary calculus is not intended for all schools nor for all teachers or all pupils in any school. It is not intended to connect in any direct way with college entrance requirements. The future college student will have ample opportunity for calculus later. The capable boy or girl who is not to have the college work ought not on that account to be prevented from learning something of the use of this powerful tool. The applications of elementary calculus to simple concrete problems are far more abundant and more interesting than those of algebra. The necessary technique is extremely simple. The subject is commonly taught in secondary schools in England, France and Germany and appropriate English texts are available.*

6. **History and Biography.** Historical and biographical

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* Quotations and typical problems from one of these texts will be found in a supplementary note appended to this report.

material should be used throughout to make the work more interesting and significant.

At the present time these topics (1–5) can probably in most high schools be given most advantageously as separate units of a two-year program. However, the National Committee is of the opinion that methods of organization are being experimentally perfected whereby teachers will be enabled to present the same material more effectively in combined courses unified by one or more central ideas.

Additional Electives.

Such additional electives as *mathematics of investment*, *shop mathematics*, *surveying and navigation*, *descriptive or projective geometry* will appropriately be offered by schools which have special needs or conditions, but it seems unwise for the National Committee to attempt to define them pending the results of further experience on the part of these schools.

Supplementary Note on the Calculus as a High-School Subject.

In connection with the recommendations concerning the calculus, such questions as the following may arise: Why should a college subject like this be added to a high school program? How can it be expected that high-school teachers will have the necessary training and attainments for teaching it? Will not the attempt to teach such a subject result in loss of thoroughness in earlier work? Will anything be gained beyond a mere smattering of the theory? Will the boy or girl ever use the information or training secured? The subsequent remarks are intended to answer such objections as these and to develop more fully the point of view of the Committee in recommending the inclusion of elementary work in the calculus in the high school program.

By the calculus we mean for the present purpose a study of *rates of change*. In nature all things change. How much do they change in a given time? How fast do they change? Do they increase or decrease? When does a changing quantity become largest or smallest? How can rates of changing quantities be compared?

These are some of the questions which lead us to study the elementary calculus. Without its essential principles these questions cannot be answered with definiteness.

The following are a few of the specific replies that might be given in answer to the questions listed at the beginning of this note: The difficulties of the college calculus lie mainly outside the boundaries of the proposed work. The elements of the subject present less difficulty than many topics now offered in advanced algebra. It is not implied that in the near future many secondary-school teachers will have any occasion to teach the elementary calculus. It is the culminating subject in a series which only relatively strong schools will complete and only then for a selected group of students. In such schools there should always be teachers competent to teach the elementary calculus here intended. No superficial study of calculus should be regarded as justifying any substantial sacrifice of thoroughness. In the judgment of the Committee the introduction of elementary calculus necessarily includes sufficient algebra and geometry to compensate for whatever diversion of time from these subjects would be implied.

The calculus of the algebraic polynominal is so simple that a boy or girl who is capable of grasping the idea of limit, of slope, and of velocity, may in a brief time gain an outlook upon the field of mechanics and other exact sciences, and acquire a fair degree of facility in using one of the most powerful tools of mathematics, together with the capacity for solving a number of interesting problems. Moreover, the fundamental ideas involved, quite aside from their technical applications, will provide valuable training in understanding and analyzing quantitative relations,—and such training is of value to everyone.

The following typical extracts from an English text intended for use in secondary schools may be quoted:

"It has been said that the calculus is that branch of mathematics which schoolboys understand and senior wranglers fail to comprehend. . . . So long as the graphic treatment and practical applications of the calculus are kept in view, the subject is an extremely easy and attractive one. Boys can be taught the subject early in their mathematical career, and there is no part of their mathematical training that they enjoy better or which opens up to them wider fields of useful exploration. . . . The phenomena must first be known practically and then studied philosophically. To reverse the order of these processes is impossible."

The text in question, after an interesting historical sketch, deals with such problems as the following:

A train is going at the rate of 40 miles an hour. Represent this graphically.

At what rate is the length of the daylight increasing or decreasing on December 31, March 26, etc.? (From tabular data.)

A cart going at the rate of 5 miles per hour passes a milestone, and 14 minutes afterwards a bicycle, going in the same direction at 12 miles an hour, passes the same milestone. Find when and where the bicycle will overtake the cart.

A man has four miles of fencing wire and wishes to fence in a rectangular piece of prairie land through which a straight river flows, the bank of the stream being utilized as one side of the enclosure. How can he do this so as to enclose as much land as possible?

A circular tin canister closed at both ends has a surface area of 100 sq. cm. Find the greatest volume it can contain.

Post office regulations prescribe that the combined length and girth of a parcel must not exceed 6 feet. Find the maximum volume of a parcel whose shape is a prism with the ends square.

A pulley is fixed 15 feet above the ground, over which passes a rope 30 feet long with one end attached to a weight which can hang freely, and the other end is held by a man at a height of 3 feet from the ground. The man walks horizontally away from beneath the pulley at the rate of 3 feet per second. Find the rate at which the weight rises when it is 10 feet above the ground.

The pressure on the surface of a lake due to the atmosphere is known to be 14 ℔ per sq. in. The pressure in the liquid x inches below the surface is known to be given by the law $dp/dx = 0.036$. Find the pressure in the liquid at a depth of 10 feet.

The arch of a bridge is parabolic in form. It is 5 feet wide at the base and 5 feet high. Find the volume of water that passes through per second in a flood when the water is rushing at the rate of 10 feet per second.

A force of 20 tons compresses the spring buffer of a railway stop through 1 inch, and the force is always proportional to the compression produced. Find the work done by a train which compresses a pair of such stops through six inches.

These may illustrate the aims and point of view of the proposed work. It will be noted that not all of them involve calculus, but those that do not lead up to it.

Comments, criticisms and suggestions for the revision of this preliminary report are urgently requested. They should be sent as soon as possible to J. W. Young, chairman, Hanover. N. H.